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ACCOUNT OF A DETERMINATION OF THE COEFFICIENTS OF EXPANSION OF THE WIRES OF THE JÄDERIN BASE-LINE APPARATUS.

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ACCOUNT OF A DETERMINATION

OF THE

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JÄDERIN BASE-LINE APPARATUS.

BY

Captain G. P. LENOX CONYNGHAM, R.E.

The method of measuring Geodetic Bases, designed by M. Jäderin, has been explained in Professional Paper No. 2 of 1899. The wires discussed in that paper were of hard-drawn steel and brass respectively, and their coefficients of expansion were

0·000,005,6 per 1° Fahrenheit in the case of steel

and 0·000,010 ,, ,, ,, brass.

The wires sent out to India however differed from the above in that an alloy of nickel and steel was substituted for the hard-drawn steel formerly employed.

The reason of the change is that this alloy has an extraordinarily small coefficient of expansion, which gives it two important advantages; *firstly*, that the error in a measurement caused by a wrong estimation of the temperature is much reduced, and, *secondly*, that the change in the relative lengths of the two wires corresponding to a given variation of temperature is greater than formerly, so that the power of measuring such variations is increased.

The value of the coefficient of expansion of this alloy was not however obtainable. In the Monthly Notices of the Royal Astronomical Society for January 1900 (No. 4, Volume LX), Professor G. H. Darwin, gave it as about one-fiftieth of that of ordinary steel. The makers of the apparatus said that it was so small as to be negligible, but made no definite statement of its amount; they were not able to state the composition of the alloy as it had not been made by them, and M. Jäderin, under whose instructions it had been obtained, was absent on an Arctic Expedition and could not be communicated with. Under these circumstances there was nothing to be done but to make a determination with such means as could be devised.

It was decided to adhere to M. Jäderin's method of stretching the wire by means of a spring-balance and measuring its length under a constant strain at different temperatures.

As a first step, a substantial thatched shed, 100 feet long, was built with its length lying nearly east and west; the northern side and the ends were left open, but removable screens were provided so that they could be closed at will. In this shed two substantial brick pillars were erected, 80 feet apart from centre to centre: their foundations were sunk to a depth of 2½ feet from the ground level, and they were isolated from surface tremors by a space 3 inches wide all round.

On these pillars were placed the two microscopes, G and H, (shown in Plate II) of the Colby Base-Line apparatus, which are used for comparing the compensation bars against the standards.

These microscopes are of excellent design and construction: they are provided with micrometers whose heads are divided into 100 divisions: the micrometer screw has 100 threads to the inch, and the distance from the diaphragm to the object-glass being about 2½ times that from the latter to the external focus, the value of one revolution of the micrometer head is about $\frac{1}{250}$ inch and that of 1 division $\frac{1}{25000}$ inch. The power of the eye-piece is such that the probable error of one intersection of a suitable object is about one division.

A pair of tripods had been made in the Mathematical Instrument Office, Calcutta, for straining the wires to the proper tension and bringing their scales into the focus of the microscopes. A glance at Plates I and II will show their construction: in the vertical direction a sliding rod and clamp give a rough adjustment which can be corrected by a screw; a traversing frame and screw give the lateral motion and a long screw, on which works a nut provided with capstan bars, pulls the wire up to the required strain.

A third microscope, of about the same magnifying power as those already mentioned, was used for reading the scale of the spring-balance, so as to ensure the measurement of the tension being of the same order of accuracy as the readings of the scales of the wires. The tripods and microscopes being erected and adjusted, we have the means of measuring the length of the wire in terms of the distance between the zeros of the microscopes.

To attach the spring-balances to the pulling screws of the tripods, the covers of each of the former were provided with a pair of horizontal trunnions which worked in holes in a gimbal ring capable of swinging about a vertical axis; the ends of this axis were held by the two ends of an iron shackle which was clamped to the pulling screw (Figure 1). The trunnions were placed at the centre of the cylindrical cover, so that it balanced on them; the friction between the connecting rod and the cover was thus minimised; with a little care contact between them could be altogether avoided*.

When the wire was being stretched by hand this plan seemed to work well, but under the microscope it was clear that there was still friction, chiefly between the index and the scale on the cover, for, when the strain was being taken, the index was seen to move spasmodically.

* Vide Professional Paper No. 2 of 1899, page 6, last line and figure 2.

Moreover it was found possible to alter the reading appreciably by slightly moving the cover and without disturbing the spring or the wire. Figure 2 is a sectional sketch from which it will be clear that slight movements of the scale under the index could take place without the strain being altered, unless the index is actually touching the scale, in which case there will be friction. To obviate this the cover and scale were altogether done away with and the arrangement shewn in Figure 3 adopted instead.

The ends of the steel spring are formed by bending up the last half turn at each end through a right angle, thus forming two semicircular hooks. To these hooks are screwed pieces of the form shown in the sketch. A long light brass tongue, *a e*, is fastened by a pivot at *e* and rests in a groove in the piece *f f* so that its upper surface and that of *f f* are flush with each other. The tongue has a scale of kilogrammes marked on it, and there is a cut, *b*, on the surface of *f f*. The V-shaped piece, *cc*, has a hole near *g* (concealed by the tongue in the sketch) to receive the hook which connects the wire with the spring: at the other end the piece *dd* has screw holes to receive pivots passing through a gimbal ring. Plate 2 shows the arrangement very clearly.

For use in the field a cover will be added to protect the spring from damp.

The tongue was graduated by hanging the spring vertically, applying weights of 9, 10 and 11 kilogrammes in turn and making a mark opposite the cut *b* in each case. This does not ensure that the strain will be exactly 10 kilogrammes when the graduation 10 coincides with the cut *b*, for the weight of the suspended parts of the spring-balance was not taken into account, but the difference will be small and so long as the strain is always the same its actual amount is not important, although it should not be widely different from 10 kilogrammes, for it was on the basis of this tension that Mr. Jäderin designed his apparatus and computed the most favourable weight of wire.

After this attachment had been made it was found that, on the tension being disturbed, oscillations of perfect evenness took place until the system finally came to rest.

With a nickel-steel wire, if the temperature be tolerably steady, the probable error of one measurement of the length proves to be about 2 divisions of the microscope. The error of one measurement includes that introduced by inaccuracy in adjusting the strain and that of the sum of the readings of the microscopes.

The wires are similar to those described by M. Jäderin but are 80 feet* (24·38 metres) instead of 25 metres in length. At each end there is a scale about 5 inches long, graduated to millimetres and numbered from the centre outwards, the graduations on the outer side of zero being marked + 10, + 20, &c., and those on the inner side — 10, — 20, &c. (Figure 4).

On each scale the distinguishing letter of the wire is also engraved.

There are three pairs of wires 80 feet long, *viz*:—

A nickel-steel	B brass
C "	D "
E "	F "

* The horizontal length is as nearly as possible 80 feet when the wires are strained with a tension of 10 kilogrammes the ends being at the same height and the temperature 62° F.

There is one pair 160 feet long, *viz*:—

G nickel-steel H brass

and one pair 320 feet long, *viz*:—

K nickel-steel L brass.

The arrangement for measuring the length of the wire at a known tension being now complete, means of heating the suspended wire and of ascertaining its temperature had to be devised.

A wooden trough 79 feet 4 inches long was first made. It consisted of 8 lengths, of which the two end ones were 9 feet 8 inches long and the others 10 feet. The internal dimensions were 6 inches in width by 10 inches in height. Each had a wooden lid which could be lifted on and off. These 8 lengths were placed on trestles (*vide* Fig. 5) and carefully aligned so that the middle line of their length should be in the vertical plane of the microscopes. They were given a slight slope upwards from centre to ends so as to follow approximately the curve of the suspended wire.

In this trough was placed a tin pipe running from end to end and back again: it was 2 inches in diameter, and was capable of being filled with water from two mouths at the centre. A rough plan of the pipe in the trough is seen in Fig. 6; *a a* are the two mouths, *b* is an exhaust pipe for drawing off the water, and there are two more similar ones below the mouths *a a*. The trough was closed by movable ends with sliding shutters, which, after the wire had been adjusted, could be pushed in so as to leave only the smallest possible space round it.

To measure the temperature eight thermometers were employed, one in each section of the trough; a glazed window was made about the middle of each section through which to view the thermometers. The latter were short, so that they could be put altogether inside the trough, and were fastened by clips to small pieces of wood which were suspended by wires through the lid in such a way that their height could be regulated. The bulbs of the thermometers were always placed at the same height as the wire. Sliding doors were made in the lids of the troughs through which the thermometers could be introduced or removed as occasion required. When the wire and the thermometers were in position the whole trough was well wrapped up in blanket so as to obviate the effects of variations in the temperature of the air and to prevent the escape of heat as far as possible.

It was supposed that the wires would be less sensitive than the thermometers to direct radiation of heat from the hot pipes, for M. Jäderin remarks that fine wires freely suspended take up almost exactly the temperature of the surrounding air, and calls to witness the fact that when a telescope is pointed at the sun the spider's web reticule is not burnt even in the focus of the object glass. In order therefore to protect the thermometers a piece of thick felt about 1 foot long and the same width as the trough was laid over the pipes opposite each window. Before actually beginning the experiments all the thermometers were carefully compared against the two standards which had recently returned from the Observatory at Kew where they had been compared against the English standards. The comparisons were made in water and included a range of about 110° F., namely from 46° to 156°.

Experiments were now begun on wire A (nickel-steel). The system of observation finally adopted, after trying several methods, was as follows:—The two observers took up their positions at the microscopes, and four thermometer readers were appointed to the charge of a pair of thermometers each.

The observer at that end at which were the improved spring-balance and the microscope for reading the tension, carefully adjusted the strain by a pulling up motion of the capstan head, called 'Ready' and, at once moving to the comparing microscope, made an intersection of the nearest graduation of the scale of the wire; at the same time the other observer did likewise. The first observer then returned to the capstan head and again adjusted the strain, but this time with a letting down motion, when satisfied he again called 'Ready' and again intersected—and so on. Usually the strain was taken six times in the following order:—Up, down, down, up, up, down. If the readings were unusually discordant more were taken. During this process, which occupied about five minutes, each thermometer reader continued to pass to and fro between the thermometers in his charge, recording the readings again and again: about 10 readings of each thermometer were thus obtained, scattered fairly evenly over the period during which the observation was going on.

The first observation was generally taken with empty pipes; when it was complete, the pipes were filled with hot water and after the lapse of about half an hour a second observation was taken; then some of the hot water was run off and cold water added so as to produce a medium temperature, every care being taken, by pouring cold water first into one mouth and then into the other, to make the average temperature of the pipes in each section of the length the same.

The results of the first series of observations, which were all made on wire A (nickel-steel) were as follows:—

W represents the length of the wire between the zeros of its scales; M the distance from zero to zero of the microscopes. M is only to be considered constant during sets of observations included in one column.

Date and Time of Observation	Temperature Fahrenheit		Divisions of G Microscope	Date and Time of Observation	Temperature Fahrenheit		Divisions of G Microscope	Date and Time of Observation	Temperature Fahrenheit		Divisions of G Microscope
1901				1901				1901			
Jan. 22				Jan. 31				Feb. 1			
h. m.	°			*h. m.*	°			*h. m.*	°		
1 50 P.M.	55·7	M = W +	1245	1 30 P.M.	67·3	M = W +	1709	0 10 P.M.	60·9	M = W +	1763
2 0 „	86·7		1092	2 45 „	103·7		1529	0 40 „	78·6		1646
3 0 „	98·4		1047	3 40 „	109·9		1501	3 3 „	104·4		1544
3 30 „	106·9		994	...	...		...	3 55 „	74·7		1718
Deduced expansion per 1° F., in terms of G microscope			4·98	...	...	...	4·78	...	...	...	4·92
Mean Expansion 4·89											

The length* of the wire is 80 feet, and the value of 1 division of G microscope is 3·4611 millionths of a foot.

Hence the coefficient of expansion from the above observations is $\frac{4 \cdot 89 \times 3 \cdot 4611}{80 \times 1{,}000{,}000}$ = 0·000,000,212 per 1° F.

That of ordinary steel wires is 0·000,005,6 of which the new value is about the $\frac{1}{26}$th part.

It will be noticed that the highest temperature reached in this series is just under 110°, although the water was made to boil before being run into the pipes. It seemed therefore that in spite of the blanket coverings there was a great loss of heat.

It was not considered sufficient to examine the expansion of the wires up to a temperature of 110° only, for the temperature of the air in India is frequently higher than that, nor is it certain that the wires do not become hotter than the air when exposed to the sun's rays, though there is good evidence that they do not become heated to nearly the same extent as more massive pieces of metal. Besides the behaviour of the thermometers had given rise to doubts as to whether they were taking up the temperature at the same rate as the wires. It was decided therefore to make a long trough capable of holding water in which to immerse both the wires and the thermometers.

In the case of the wooden trough and hot water pipes the scales of the wires protruded to the outside through the adjustable apertures mentioned above: in the case of the water trough however scales and all had to be inside, the trough was therefore made a little over 81 feet long. It was of galvanised iron; 2 inches wide by 10 inches deep. Bent pieces of iron were used to pass the strain from the wire inside to the spring outside the trough (Figure 5). As it would have been difficult to make water-tight windows in the sides, the short thermometers hitherto used were exchanged for long ones which were suspended vertically in such a way that their bulbs were close beside the wire, and immersed to the same depth; they could then generally be read at once, though occasionally, at the lower temperatures, they had to be lifted for a moment.

The trough had no lid, but after the wire had been suspended and the thermometers adjusted the whole was carefully wrapped up in blanket, the smallest possible aperture being left round each thermometer.

It was found that the steam rising from the hot water condensed on the object-glasses of the microscopes and obscured them. To prevent this, an arrangement was made by which the lower surface of a piece of plate-glass could be brought into contact with the water just below the microscope, and all steam which rose to either side of it and might have found its way to the object-glass was caught by suitable attached pieces of cloth (*vide* Plate II). The device answered its purpose satisfactorily.

The new set of thermometers, which were of very good quality, were carefully compared against the standards and corrections deduced.

The horizontal length between the ends of a wire suspended in water is not the same as when it is suspended in air, for immersion in a denser fluid causes diminution of sag, but this does not vitiate a determination of the coefficient of expansion based on observations of the change in the horizontal length due to change of temperature.

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* This is the horizontal length, the actual length is greater by about $\frac{1}{8}$ of an inch.

All the eighty-feet wires were examined in the water trough and the following table shows the results obtained:—

NICKEL-STEEL WIRES.

Date and Time of Observation		Wire	Temperature Fahrenheit		Divisions of G Microscope	Expansion per 1° F. in terms of G Microscope	Coefficient of Expansion
1901	h. m.		°			d	
Mar. 18	1 25 P.M.	A	128·1	M = W +	1015	6·44	0·000,000,279
	2 40 "		101·4		1187	6·07	263
	4 0 "		72·1		1355	5·73	248
Mar. 19	0 15 P.M.	A	71·1		1431	5·51	238
	1 5 "		109·2		1221	4·62	200
	2 50 "		156·0		1039	3·89	168
Mar. 14	0 30 P.M.	C	67·5	M = W +	1676	...	......
	1 30 "		135·6		1308	5·40	234
Mar. 15	0 30 P.M.	C	135·5		1197	8·13	352
	1 45 "		97·5		1506	7·01	303
	3 15 "		69·9		1657	5·47	237
Apr. 11	0 0 NOON	E	72·8	M = W +	1741	2·82	122
	0 50 P.M.		144·7		1538	3·55	154
	2 30 "		112·2		1601	1·94	084
					Mean ...	5·12	0·000,000,222
							± 14

BRASS WIRES.

Date and Time of Observation		Wire	Temperature Fahrenheit		Divisions of G Microscope	Expansion per 1° F. in terms of G Microscope	Coefficient of Expansion
1901	h. m.		°			d	
Mar. 12	1 30 P.M.	B	68·3	M = W +	11934	...	......
	3 30 „		112·4		1032	247·21	0·000,010,695
Mar. 13	1 45 P.M.	B	121·4		1108	...	......
	3 30 „		71·5		13768	253·71	10,976
Apr. 3	2 25 P.M.	B	71·8		22161	...	......
	3 15 „		154·7		1365	250·86	10,853
Mar. 20	0 10 P.M.	D	71·4	M = W +	22504	...	......
	1 0 „		122·3		9741	250·75	10,848
	2 50 „		153·7		1548	260·92	11,288
	3 40 „		73·6		21642	250·86	10,853
Mar. 21	1 30 P.M.	D	74·8		17125	...	......
	2 40 „		142·4		576	244·81	10,591
Apr. 12	0 15 P.M.	F	71·5	M = W +	22312	250·20	10,825
	0 55 „		152·9		1946	251·02	10,860
	1 15 „		118·5		10514	249·07	10,776
					Mean ...	250·94	0·000,010,856 ± 39

Even the largest changes in the length of the nickel-steel wires were well within the compass of the comparing microscopes; but with the brass wires this was by no means the case, for the expansion between extreme temperatures exceeded 20,000 divisions, or $\frac{1}{8}$ of an inch. To measure these changes recourse was had to the scales on the wires, both the number of the graduation intersected and the microscope reading being recorded. Thus in the case of the third and fourth measurements of wire D the readings were as follows:—

Temperature Fahrenheit	Graduation Intersected	Microscope Reading
153°·7	− 6	+ 1547^{d}·8
73 ·6	+ 16	+ 784 ·8

The actual length of each millimetre on the scale from − 6 to + 16 was then measured with the microscope and the sum used in reducing the two observations to comparable terms. The results in this instance were:—

	d		d		d		d		d
+ 16 to + 15	941	+ 11 to + 10	951	+ 6 to + 5	951	+ 1 to 0	946	− 4 to − 5	938
15 „ 14	949	10 „ 9	933	5 „ 4	953	0 „ − 1	954	5 „ 6	953
14 „ 13	948	9 „ 8	954	4 „ 3	946	− 1 „ 2	947	∴ 22^{mm} = 20857	
13 „ 12	953	8 „ 7	950	3 „ 2	944	2 „ 3	950	1^{mm} = 948·05	
12 „ 11	948	7 „ 6	944	2 „ 1	946	3 „ 4	958		

By comparison against the divisions of the standard foot the value of 1 division of G microscope was found to be 1·1537 millionths of a yard; also 1 yard = 914·391792 millimetres. Whence we find one millimetre = 947·96 divisions of G microscope.

The accordance between the two values of one millimetre in terms of G microscope is surprising.

It was not to be expected that the length of the individual millimetres would agree very well together, for the graduations are cut for reading by the naked eye and are of course unsuitable for intersection under a powerful microscope.

The scales of all the wires were examined in the same way. They appear to be of uniform quality, both as regards the mean length of their divisions and the equality of one division with another.

This concluded the determinations of the expansions of the wires. The equations derived from the observations of the brass wires were treated by minimum squares in order to see whether there was any evidence in favour of inserting a term in the expression for the expansion containing the square of the temperature.

The resulting expression was

$$255\cdot5\,(T - t) - 0\cdot08\,(T - t)^2 = E$$ (the expansion of a brass wire 80 feet long).

The second term is much more likely to be due to observational errors than to any real cause. An inspection is sufficient to show that in the case of the nickel-steel wires also errors of observation are too great to allow of a second term being revealed, the mean values of the expansions given above have therefore been adopted. They are here repeated.

Nature of Wire	Per 1° Fahrenheit		
	Coefficient of Expansion	Expansion of 80 feet Wire	
		Division of G Microscope	Millimetres
Nickel and steel alloy ...	0·000,000,222	5·12	0·00540
Brass	0·000,010,856	250·94	0·26472

Before dismantling the apparatus the opportunity was taken to measure a length of 80 feet and to compare each of the wires against it.

The measurement was effected by means of the Indian Standard 10 feet Bar A (wrought iron), in terms of which all the measurements of the Survey of India are expressed.

Between the massive brick pillars which were built for the comparing microscopes seven smaller pillars were now built dividing the total interval of 80 feet into equal portions of 10 feet. Trestles with camels belonging to the Colby Apparatus (*vide* Professional Paper No. 3 of 1900 Figs. 1 and 4') were set out for the reception of the bar, and two portable microscopes, K and L, were placed on the first two of the new pillars. The boning instrument was set up in the prolongation of the line of G and H microscopes. The bar was now placed on the first two camels and one of its dots brought into focus under G; then its direction was made parallel to the line of the microscopes, and it was levelled. K microscope was now adjusted over the forward dot of the bar, until the focus was correct and the cross wires truly intersected it; then the bar was carefully lifted from under G and K and carried forward to the next pair of trestles, when the rear dot was placed in the focus of K so that it appeared in the intersection of its cross wires, the microscope itself not being disturbed in any way. The bar was next aligned and levelled, any consequent movement of the dot under K being corrected by means of the camel. Then L was placed over the forward dot, — and so on until the H microscope was reached.

Bar A contains two wells for the bulbs of thermometers*. On each occasion as soon as the observers were satisfied with the adjustment of the bar under the microscopes, they at once moved to the thermometers and recorded their readings.

The microscopes K and L are old, very shaky and of inconvenient construction; this rendered the measurement somewhat tedious in execution, and greatly interfered with the precision of the results.

* The thermometers, known as α and β, were carefully compared against the standards before the measurement.

Four measurements of the 80 feet length were made and two sets of comparisons of the wires against it, in the following order:—

Measurement	April	22, 23, 1901
Comparison	„	24, 25
Measurement	„	25
Comparison	„	29
Measurement	„	29

Throughout the period neither G nor H microscope was touched, except to make an intersection with the micrometer.

RESULTS OF THE MEASUREMENTS.

				Divisions of G microscope
1901 April	22	Length between zeros of microscopes	= 8 **A**	+ 3042·5
„	23	„	„	+ 2993·1
„	25	„	„	+ 2994·2
„	29	„	„	+ 3054·6
			Mean = 8 **A**	+ 3021·1

the symbol **A** denoting the length of the 10-foot standard bar **A** at a temperature of 62° Fah.

The greatest difference from the mean $= \overset{G}{33 \cdot 5} = \overset{mm}{0 \cdot 035}$ or $\frac{1}{690,000}$ of the length.

Before the comparisons were made all the wires had been hanging up under a tension of 5 or 6 kilogrammes for several weeks*. Between the comparisons wire **A** was rolled on a drum about 18 inches in diameter and kept so, until the moment before the second comparison; all the other wires were hung up as usual. For the comparisons the iron trough was removed but the wooden one remained, and the wires and thermometers were suspended in it, so as to be protected from currents of air.

In reducing to a temperature of 62° Fahrenheit the values of the expansions derived from these experiments were made use of. The results were as follows:—

Wire	Date	Length of Wire at 62° Fahrenheit	Date	Length of Wire at 62° Fahrenheit	Change
A Nickel-Steel ...	1901 April 24	mm 8 **A** − 1·293	1901 April 29	mm 8 **A** − 1·340	mm − 0·047
B Brass ...	„ „	− 1·789	„ „	− 1·810	− 0·021
C Nickel-Steel ...	„ „	− 0·425	„ „	− 0·559	− 0·134
D Brass ...	„ „	+ 0·349	„ „	+ 0·157	− 0·192
E Nickel-Steel ...	„ 25	+ 0·268	„ „	+ 0·274	+ 0·006
F Brass ...	„ „	− 0·363	„ „	− 0·425	− 0·062

* In Plate I three of them may be distinguished hanging above the observers' heads.

During the comparisons of the brass wires the variability of the readings was very great, showing that the temperature was far from steady; the steel wires however are so little sensitive to change of temperature that the individual intersections agreed well together. If therefore the changes in the measured lengths were due to errors of observation or to a difference between the real temperature of the wire and that recorded by the thermometers, one would have expected these changes to be distinctly larger and less constant in sign in the case of the brass wires than in that of the steel ones. This however is not found to be the case.

With the exception of wire E all the wires seemed to be shorter on the 29th than on the former occasion. This would have inclined one to believe that there had been a change in the length of the standard, but M. Jäderin found the same peculiarity and was forced after much hesitation to admit "the very improbable supposition that all wires altered in an identical way." The evidence before us on this subject is of course by no means conclusive, not even strong indeed, for in the case of wires A, B, E and F the change is not greater than the difference between individual measurements of the standard, and it is impossible to say with regard to the latter whether a change actually took place or whether the difference is merely due to errors of measurement.

It is satisfactory to note that the rolling up and unrolling of wire A does not seem to have affected its length.

A final set of experiments were made in order to examine the behaviour of the wires and thermometers under the conditions which prevailed during the first series of observations of the expansion of wire A.

The expansion of a brass wire per 1° Fahrenheit being about 250 divisions of G microscope (*i.e.* about ¼ millimetre), variations of its temperature when inside the trough containing the pipes will be very accurately measured by the changes in its length. Therefore by reversing the former process and using the wire to determine the temperature inside the trough, the trustworthiness of the indications of the thermometers may be examined.

The wire and the thermometers were arranged as for a measurement, the pipes being empty, and were left undisturbed all night. The next day the length of the wire was measured and the thermometers read, and it was *assumed* that the mean thermometer reading indicated the mean temperature of the wire; then hot water was run in, a second measurement was made and a set of readings was taken as soon as possible. From the wire's length the increase of temperature was computed and thence the error of the mean thermometer reading. A considerable interval was next allowed to elapse, without any alteration of the conditions, and then another similar set of observations was taken, and so on. The steps taken are sufficiently explained in the tables of results.

Observations were taken under three different conditions:—

(1). The pipes bare, except for a piece of felt about 1 foot long placed under each thermometer. (The same conditions as in the first set of observations of wire A, before the water trough was made).

(2). The pipes altogether bare,—wire and thermometers equally exposed to direct radiation.

(3). The pipes covered with blanket throughout their length,—equal protection from radiation for wire and thermometers.

TABLE I.

Conditions as in (1).

Remarks	Date	Time	Temperature by Thermometers Fahrenheit	Temperature by Wire	Difference
	1901	P.M. h m	°	°	°
Pipes empty ...	May 2	1 0	86·2	(Assumed) 86·2	0·0
Hot water run in ...		1 25	117·9	129·5	−11·6
No change ...		2 30	118·1	124·9	− 6·8
Cold water added ...		4 10	91·7	93·3	− 1·6
Left all night ...	„ 3	0 25	84·2	84·1	+ 0·1
Hot water added ...		1 20	113·8	122·9	− 9·1
No change ...		2 30	112·8	118·8	− 6·0
No change ...		4 0	109·5	113·0	− 3·5

TABLE II.

Conditions as in (2). Pipes altogether bare.

Remarks	Date	Time	Temperature by Thermometers Fahrenheit	Temperature by Wire	Difference
	1901	P.M. h m	°	°	°
Pipes empty ...	May 6	0 10	81·0	(Assumed) 81·0	0·0
Hot water run in ...		1 30	120·3	128·3	− 8·0
No change ...		2 50	113·9	119·2	− 5·3
No change ...		3 45	110·4	115·4	− 5·1
Cold water added ...		4 0	95·3	97·8	− 2·5

TABLE III.

Conditions as in (3). Pipes covered.

Remarks	Date	Time	Temperature by Thermometers Fahrenheit	Temperature by Wire	Difference
	1901	P.M. h m	°	°	°
Pipes empty ...	May 7	0 15	79·5	(Assumed) 79·5	0·0
Hot water added ...		1 30	103·6	104·1	− 0·5
No change ...		2 55	102·7	103·8	− 1·1
Cold water added ...		3 50	88·5	88·7	− 0·2

It is clear from the above tables that the expectation which led to the use of the protecting pieces of felt below the thermometers, namely that the latter would be more affected than the wire by direct radiation from the hot pipes, was not well founded. In every case the wire became hotter than the thermometers. The difference when the pipes were uncovered is very large; with the blanket over the pipes there is a much better agreement, but the range of temperature is greatly reduced.

Three different patterns of thermometers were in use during these experiments: two were of the short kind as used in the first series of determinations of the expansion, one was an unmounted thermometer with the graduations and numbers marked on the glass, and the remaining five were attached to metal mounts, the graduations being on the glass, but the numbers on the metal.

It was noticed that the readings of the last kind were systematically lower than those of the others by from 2° to 4°, when there was hot water in the pipes. The removal of the metal scales caused this difference to disappear and it seems that the upper part of the scale, being in the cooler outside air, was able to conduct away the heat from the neighbourhood of the bulb to such an extent as to produce the large differences that were noticed. When the thermometers were immersed in water no such difference was observable. In the above experiments the thermometer readings are those obtained after the removal of the metal scales.

The effect of the presence of the metal scale is mentioned as being possibly useful to other experimenters. If the want of a sufficient number of thermometers of one kind had not led to the use of different patterns this source of error would certainly not have been thought of.

G. P. LENOX CONYNGHAM.

Fig. 1 (not to scale)

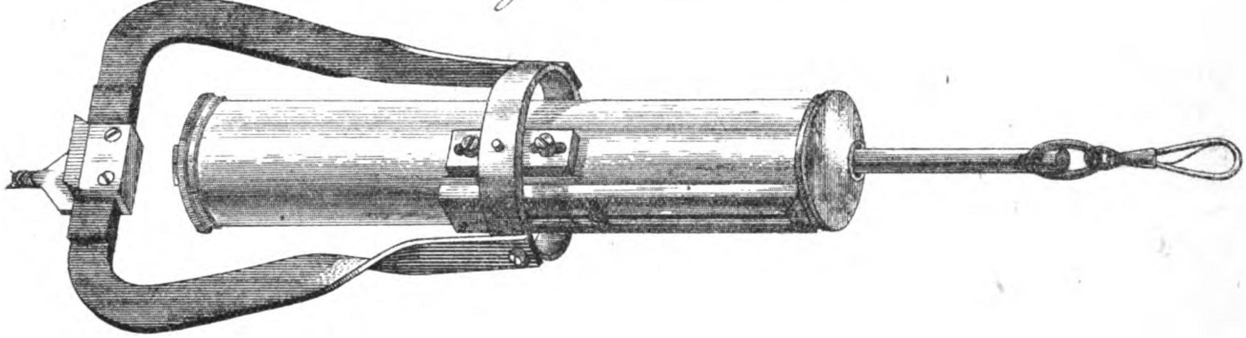

Fig. 2 (not to scale)

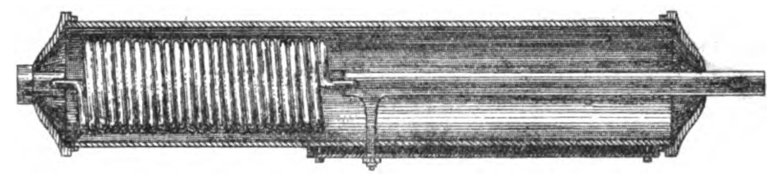

Fig. 3 (not to scale)

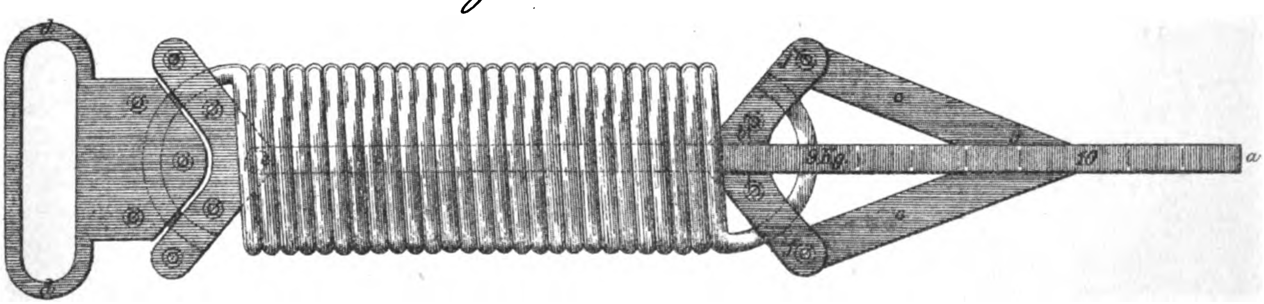

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Fig. 4 (full size)

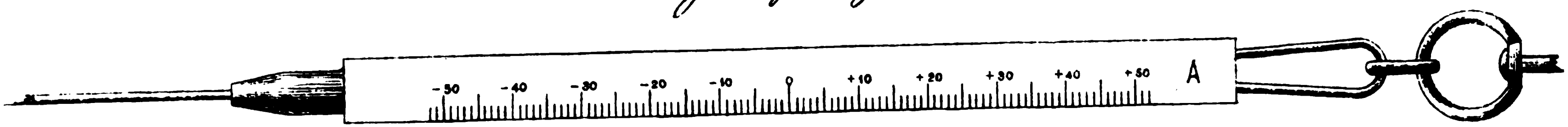


Fig. 5 (not to scale)

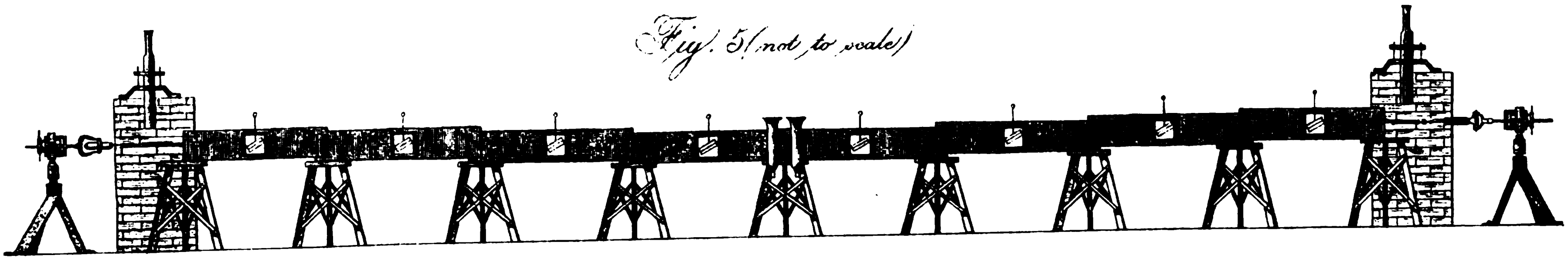
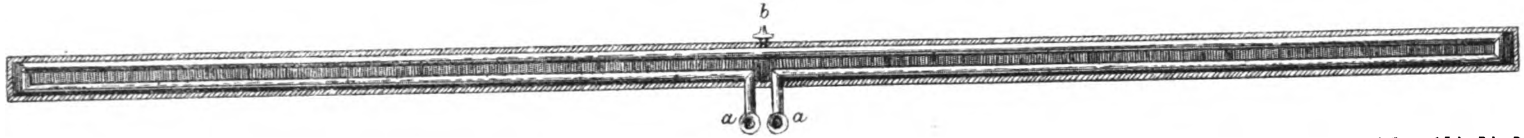


Fig. 6 (not to scale)



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